

Chapitre 4 – Pour reprendre contact – Réponse exercice 3

$$\text{a. } U_n = 1 + 2 + 2^2 + \dots + 2^n = \frac{1-2^{n+1}}{1-2} = 2^{n+1} - 1$$

$$\text{b. } V_n = \sum_{k=1}^n 2k + \sum_{k=1}^n 3 = 2 \sum_{k=1}^n k + 3n = 2 \frac{n(n+1)}{2} + 3n = n^2 + 4n$$

$$\text{c. } W_n = 1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \dots + \left(\frac{1}{3}\right)^n = \frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \frac{1}{3}} = \frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n+1}\right)$$

$$\text{d. } T_n = \sum_{k=0}^n -2k + \sum_{k=0}^n 3 \times 5^k = -2 \sum_{k=0}^n k + 3 \sum_{k=0}^n 5^k.$$

$$\text{Or } \sum_{k=0}^n k = \sum_{k=1}^n k = \frac{n(n+1)}{2} \text{ et}$$

$$\sum_{k=0}^n 5^k = 1 + 5 + \dots + 5^n = \frac{1-5^{n+1}}{1-5} = \frac{1}{4}(5^{n+1} - 1).$$

$$\text{Donc } T_n = -n(n+1) + \frac{3}{4}(5^{n+1} - 1).$$